SPECIAL FEATURES OF MEASUREMENT OF NONSTATIONARY HEAT FLUXES BY HEAT METERS IMPLEMENTING THE AUXILIARY WALL METHOD

G. N. Dul'nev, V. A. Kuz'min,
N. V. Pilipenko, and S. V. Tikhonov

UDC 536.629.7

The feasibility of measuring nonstationary heat fluxes with normal heat meters is analyzed. Calculation correlations are given for determining the variable flux from the measured drop in temperature through the thickness of the heat meter.

Heat meters of the auxiliary wall type have become very popular for measuring stationary heat fluxes [1]. The simplicity of flux determination and advances in sensor manufacturing techniques and measurement procedures in recent years have encouraged researchers to use this method to determine nonstationary fluxes [2]. There is, however, a considerable limitation on the widespread use of these heat meters — a large error, especially in determinations of rapidly fluctuating fluxes [3]. This error is fundamental and is related to a shortage of information obtained experimentally. The physical properties of the device (heat-meter constant) and the drop in temperature through its thickness at various moments in time are usually known.

This statement will be examined in more detail for two common heat-meter models — a single plate and a system of bodies of type plate–half-space.

Unbounded Plate Model (Fig. 1a)

Let the surface x = 0 of an unbounded plate receive a heat flux q. Let us assume that the thermophysical properties of the heat meter and the conditions for heat exchange at the boundary are not dependent on temperature. In the case of q = const the temperature field of the plate is linear:

$$t = C_1 x + C_2.$$

If the temperatures of the surfaces $t(0) = t_1$ and $t(\delta) = t_2$ are known, then the relationship between the flux and the temperature drop $\Delta t = t_1 - t_2$ takes the form

$$q = \frac{\lambda}{\delta} \Delta t. \tag{1}$$

From this very simple example it is clear that, although we must know two temperatures in order to determine C_1 and C_2, subsequent transformations make it possible to convert to the difference Δt. This conversion is only possible due to the linear distribution of the temperatures, when the value of the flux is the same in any cross section of the plate.

Let us solve this problem for a variable flux $q(\tau)$. The temperature field of the plate is described by the equation

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{a} \cdot \frac{\partial t}{\partial \tau}, \tag{2}$$

Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 5, pp. 772-778, May, 1977. Original article submitted May 20, 1976.

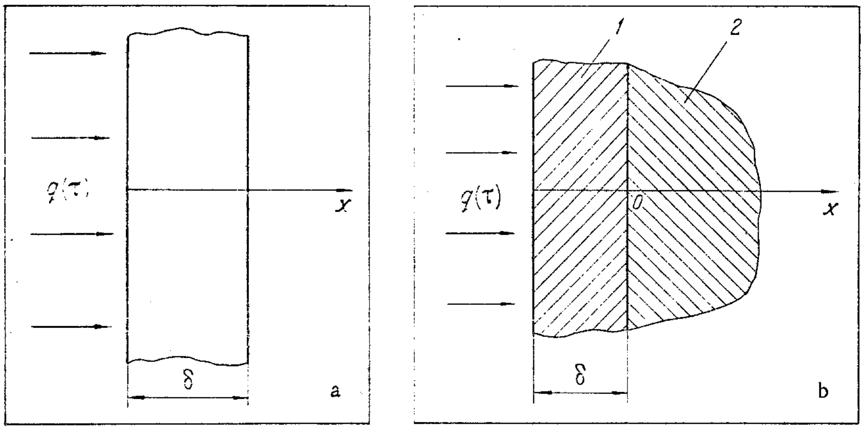

Fig. 1. Heat-meter models: type 1 (a) and type 2 (b).

with edge and initial conditions

$$t(0,\ \tau) = t_1(\tau),\quad t(\delta,\ \tau) = t_2(\tau),\quad t(x,\ 0) = t_0.$$

The solution to problem (2) is obtained by analogy with [7]:

$$t(\bar{x},\ \mathrm{Fo}) = 2\int\limits_0^{\mathrm{Fo}} \sum_{n=1}^{\infty} [t_1(\mathrm{Fo}_*) - (-1)^n t_2(\mathrm{Fo}_*)]\, \mu_n \sin \mu_n \bar{x}\, \exp[-\mu_n^2(\mathrm{Fo} - \mathrm{Fo}_*)]\, d\mathrm{Fo}_* + t_1(\mathrm{Fo})$$

$$- \bar{x}[t_2(\mathrm{Fo}) - t_1(\mathrm{Fo})] - 2\sum_{n=1}^{\infty} [t_1(\mathrm{Fo}) - (-1)^n t_2(\mathrm{Fo})] \frac{\sin \mu_n \bar{x}}{\mu_n}. \tag{3}$$

Here

$$\bar{x} = \frac{x}{\delta};\quad \mu_n = n\pi,\quad n = 1,\ 2,\ \ldots;\quad \mathrm{Fo} = \frac{a\tau}{\delta^2}.$$

The heat flux passing through the sensor is determined by Fourier's law

$$q(\mathrm{Fo}) = -\frac{\lambda}{\delta} \cdot \left.\frac{\partial t}{\partial \bar{x}}\right|_{\bar{x}=0}.$$

Taking into account (3), we find

$$q(\mathrm{Fo}) = -\frac{\lambda}{\delta}\left\{2\sum_{n=1}^{\infty} [t_1(0) - (-1)^n t_2(0)]\right.$$

$$\times \exp(-\mu_n^2 \mathrm{Fo}) - 2\int\limits_0^{\mathrm{Fo}} \sum_{n=1}^{\infty} \left[\frac{dt_1}{d\mathrm{Fo}} - (-1)^n \frac{dt_2}{d\mathrm{Fo}_*}\right]$$

$$\left.\times \exp[-\mu_n^2(\mathrm{Fo} - \mathrm{Fo}_*)]\, d\mathrm{Fo}_* + t_2(\mathrm{Fo}) - t_1(\mathrm{Fo})\right\}. \tag{4}$$

From (4) it follows that in the case of q = const a stationary state $t_1(\mathrm{Fo}) \equiv t_1$, $t_2(\mathrm{Fo}) \equiv t_2$ is established in the heat meter over a certain period of time and formula (4) is converted into (1).

For q = var and under the condition that an adequate time has passed from the beginning of the experiment and that the rates $dt_1/d\mathrm{Fo}$ and $dt_2/d\mathrm{Fo}$ are low, the series and the integral on the right-hand side of (3) can be neglected and a relationship structurally similar to (1) can be used:

$$q(\mathrm{Fo}) = -\frac{\lambda}{\delta}[t_2(\mathrm{Fo}) - t_1(\mathrm{Fo})] = \frac{\lambda}{\delta}\Delta t(\mathrm{Fo}). \tag{5}$$

The admissibility of using (4) to determine nonstationary fluxes is governed by the error involved in the transition from (3) to (4) and, in general, it must be investigated separately.

Recommendations on using formulas analogous to (5) are encountered in previously published literature. For example, as shown in [2], calculations according to the formula

$$q(\tau) = \frac{\lambda}{\delta}\left[t|_{x=0}\left(\tau + \frac{\delta^2}{3a}\right) - t|_{x=\delta}\left(\tau - \frac{\delta^2}{6a}\right)\right] \tag{6}$$

provide satisfactory results for the condition that $t(0, \tau)$ and $t(\delta, \tau)$ do not contain frequencies greater than $0.1\ a/\delta^2$. Although these recommendations do indicate the area in which formulas of the same form as (5) can be used, nevertheless the dependences given are partial and in each case additional grounds must be found for the feasibility of using them.

Thus, for determining nonstationary fluxes on the basis of the heat-meter model under examination there is, in general, inadequate information only on the conductivity and the drop in temperature through the thickness of the heat meter.

Half-Space Plate Model (Fig. 1b)

A model of this type is examined in [1] for the case of q = const. We shall solve this problem for a variable heat flux.

The temperature fields of the heat meter $t_1(x, \tau)$ and the base $t_2(x, \tau)$ are described by the equations

$$\frac{\partial t_i}{\partial \tau} = a_i\left(\frac{\partial^2 t_i}{\partial x^2}\right), \quad i = 1,\ 2. \tag{7}$$

The $x = -\delta$ surface absorbs the heat flux

$$q(\tau) = -\lambda_1 \left.\frac{\partial t_1}{\partial x}\right|_{x=-\delta}, \tag{8}$$

the magnitude of which must be determined. The other boundary and initial conditions take the following form:

$$\left.\frac{\partial t_2}{\partial x}\right|_{x\to\infty} = 0 \quad \text{or} \quad t_2|_{x\to\infty} = \text{const}; \tag{9}$$

$$\lambda_1 \left.\frac{\partial t_1}{\partial x}\right|_{x=0} = \lambda_2 \left.\frac{\partial t_2}{\partial x}\right|_{x=0}, \quad t_1|_{x=0} = t_2|_{x=0}; \quad t_i|_{\tau=0} = t_c, \quad i = 1,\ 2. \tag{10}$$

It should be noted that when the problem is formulated the following assumptions are made: the thermophysical parameters are not dependent on temperature; an ideal thermal contact occurs between the bodies.

The solution to the problem posed can be written in transforms as [1]

$$\Delta\theta(s) = Y_q(s)\,Q(s), \tag{11}$$

$$Y_q(s) = \frac{\sqrt{a_1}}{\lambda_1}\cdot\frac{\operatorname{ch}\sqrt{\frac{s}{a_1}}\,\delta + \varkappa \operatorname{sh}\sqrt{\frac{s}{a_1}}\,\delta - 1}{\sqrt{s}\left(\operatorname{sh}\sqrt{\frac{s}{a_1}}\,\delta + \varkappa \operatorname{ch}\sqrt{\frac{s}{a_1}}\,\delta\right)}; \tag{12}$$

hence, the transform of the flux being sought

$$Q(s) = \frac{1}{Y_q(s)}\,\Delta\theta(s). \tag{13}$$

In the region of the inverse transforms, expression (13) is matched by the convolution of functions which are, in turn, the inverse transforms of the expressions $1/Y_q(s)$ and $\Delta\theta(s)$.

Analyzing (13), it is easy to show that $1/Y_q(s)$ has an order of magnitude $\sqrt{s}$ and thus additional transforms of (13) are necessary. The next scheme for solving the problem uses [6] for first-order Volterra integral equations.

We introduce the quantities

$$k(\tau) = \int_0^\tau q(\tau)\,d\tau \quad \text{and} \quad q(\tau) = \frac{dk(\tau)}{d\tau}, \tag{14}$$

which are equivalent. According to [5], the transform of the $k(\tau)$ function is related to $Q(s)$ by the relation $Q(s) = K(s)s$, which allows us to rewrite (13) in another form:

$$K(s) = \frac{1}{sY_q(s)}\,\Delta\theta(s). \tag{15}$$

Expression (15) satisfies the condition for the inverse Laplace transform:

$$k(\tau) = \int_0^\tau \varphi(\xi)\,\Delta t(\tau-\xi)\,d\xi. \tag{16}$$

The flux being sought $q(\tau)$ can be obtained using the Leibniz rule [7]:

$$q(\tau) = \int_0^\tau \varphi(\xi)\,\frac{d[\Delta t(\tau-\xi)]}{d\tau}\,d\xi \tag{17}$$

for $\Delta t(0) = 0$. Using the property of the convolution

$$q(\tau) = \int_0^\tau \varphi(\tau-\xi)\,\frac{dt(\xi)}{d\xi}\,d\xi, \tag{18}$$

and then, in order to complete the solution, we must find the form of the $\varphi(\xi)$ function, which is the inverse transform of the expression

$$F(s) = \frac{1}{sY_q(s)} = \frac{\lambda_1}{a_1}\cdot\frac{\operatorname{sh}A\sqrt{s}+\varkappa\operatorname{ch}A\sqrt{s}}{\sqrt{s}(\varkappa\operatorname{sh}A\sqrt{s}+\operatorname{ch}A\sqrt{s}-1)}, \tag{19}$$

$$A = \frac{\delta}{\sqrt{a_1}}, \quad \varkappa = \frac{\lambda_2}{\lambda_1}\sqrt{\frac{a_1}{a_2}}.$$

It is not always possible to establish in a general case the exact values of the $\varphi(\xi)$ function. Here we shall give solutions for several special cases of practical interest.

1. The body 2 on which the heat meter 1 is placed is a heat insulator ($\lambda_1 \gg \lambda_2$, $\varkappa \to 0$). In this case,

$$F(s) = \frac{\lambda_1}{\sqrt{a_1}}\cdot\frac{\operatorname{sh}A\sqrt{s}}{\sqrt{s}(\operatorname{ch}A\sqrt{s}-1)}, \tag{20}$$

and having used the Vashchenko–Zakharchenko theorem we obtain [7]

$$\varphi(\xi) = \frac{2\lambda_1}{\delta}\sum_{n=0}^{\infty}\exp\left(-\frac{4\pi^2n^2}{A^2}\,\xi\right). \tag{21}$$

Taking into account (21), the expression for the heat flux will take the form

$$q(\tau) = \frac{2\lambda_1}{\delta}\,\Delta t(\tau)\sum_{n=0}^{\infty}\exp\left(-\frac{4\pi^2n^2}{A^2}\,\tau\right) - \frac{8\pi^2\lambda_1 a_1}{\delta^3}$$

$$\times \int_0^{\tau} \left\{ [\Delta t(\xi) - \Delta t(\tau)] \sum_{n=1}^{\infty} n^2 \exp\left[-\frac{4\pi^2 n^2}{A^2}(\tau - \xi) \right] \right\} d\xi. \tag{22}$$

2. The body 2 is an ideal conductor ($\lambda_1 << \lambda_2$, $\varkappa \to \infty$). Then by analogy with Para. 1,

$$q(\tau) = \frac{\lambda_1}{\sqrt{a_1}} \Delta t(\tau) \left[\frac{1}{A} + \frac{2}{A} \sum_{n=1}^{\infty} \exp\left(-\frac{\pi^2 n^2}{A^2} \tau \right) \right]$$

$$- \frac{2a_1 \lambda_1 \pi^2}{\delta^3} \int_0^{\tau} \left\{ [\Delta t(\xi) - \Delta t(\tau)] \sum_{n=1}^{\infty} n^2 \exp\left[-\frac{\pi^2 n^2}{A^2}(\tau - \xi) \right] \right\} d\xi. \tag{23}$$

3. The thermophysical properties of the heat meter and the base coincide ($\varkappa = 1$):

$$q(\tau) = \frac{\lambda_1}{\sqrt{\pi a_1 \tau}} \Delta t(\tau) \left[1 + \sum_{n=1}^{\infty} \exp\left(-\frac{A^2 n^2}{4\xi} \right) \right]$$

$$- \frac{\lambda_1}{\sqrt{\pi a_1}} \int_0^{\tau} \left\{ \frac{1}{2(\tau - \xi)^{3/2}} \left[1 + \sum_{n=1}^{\infty} \exp\left[-\frac{A^2 n^2}{4(\tau - \xi)} \right] \right. \right.$$

$$\left. \left. - \frac{A^2}{4(\tau - \xi)^{5/2}} \sum_{n=1}^{\infty} n^2 \exp\left[-\frac{A^2 n^2}{4(\tau - \xi)} \right] \right] \right\} [\Delta t(\xi) - \Delta t(\tau)] \, d\xi. \tag{24}$$

Dependences (22)-(24) are not suitable for practical use, since in experiments discrete values are usually obtained for the measured quantities (Δt) and the form of the functional dependence $\Delta t(\tau)$ must be known in order to use the formulas indicated. Calculation correlations can be obtained by replacing the integrals in (22)-(24) by sums. This does not present great difficulties and, for example, for the latter case the calculation formulas will take the form

$$q(\tau) = \frac{\lambda_1}{\sqrt{\pi a_1 \tau}} \Delta t(\tau) \left[1 + \sum_{n=1}^{\infty} \exp\left(-\frac{A^2 n^2}{4\tau} \right) \right] - \frac{\lambda_1}{\sqrt{\pi a_1}} \sum_{k=1}^{n} \left[\frac{\sqrt{\Delta\tau}}{3} [(N - k + 1)^{3/2} - (N - k)^{3/2}] \right.$$

$$\times \left\{ \frac{1}{N\Delta\tau - k\Delta\tau} - \frac{1}{N\Delta\tau - (k-1)\Delta\tau} - \sum_{n=1}^{\infty} \frac{4}{A^2 n^2} \left\{ \exp\left[-\frac{A^2 n^2}{4(N\Delta\tau - k\Delta\tau)} \right] \right. \right.$$

$$\left. \left. - \exp\left[-\frac{A^2 n^2}{4[N\Delta\tau - (k-1)\Delta\tau]} \right] \right\} \right\} - \frac{2}{\sqrt{\Delta\tau}} [\sqrt{N - k + 1} - \sqrt{N - k}]$$

$$\left. \times \sum_{n=1}^{\infty} \left\{ \exp\left[-\frac{A^2 n^2}{4(N\Delta\tau - k\Delta\tau)} \right] - \exp\left[-\frac{A^2 n^2}{4\Delta\tau(n - k + 1)} \right] \right\} \right]; \quad \tau = N\Delta\tau. \tag{25}$$

Thus, the properties of the heat meter (δ, λ_1, a_1), the drop in temperature at various moments in time, and the thermophysical properties of the half-space $\varkappa = (\lambda_2/\lambda_1)\sqrt{(a_1/a_2)}$ must be known in order to measure nonstationary fluxes using a heat meter fastened onto a semibounded body. As yet, however, this problem can be solved only for special cases $\varkappa = 0, 1, \infty$. It should be noted also that working formulas of the type in (25) for calculating a flux are so cumbersome that it is best to use a computer for practical applications.

NOTATION

q, heat flux density; τ, time; t_1, t_2, temperatures of heat meter and base; Δt, temperature drop through thickness of heat meter; λ_1, λ_2, a_1, a_2, thermal conductivities and thermal diffusivities of heat meter (1) and base (2), respectively; δ, thickness of heat meter; x, current coordinate; $Q(s)$, $\Delta\theta(s)$, Laplace transform of heat flux $q(\tau)$ and temperature drop $\Delta t(\tau)$; s, Laplace transform parameter; $Y_q(s)$, transfer function from heat flux to temperature drop; N, number of sections into which time interval $0 - \tau$ is divided.

LITERATURE CITED

1. O. A. Gerashchenko, Principles of Heat Measurement [in Russian], Naukova Dumka (1971).
2. B. Douglas, Instrum. Contr. Syst., 45, No. 5 (1975).
3. G. N. Dul'nev and N. V. Pilipenko, Inzh.-Fiz. Zh., 29, No. 5 (1975).
4. G. Doetsch, Guide to the Application of Laplace and Z Forms, 2nd ed., Van Nostrand Reinhold.
5. M. L. Krasnov, Integral Equations [in Russian], Nauka, Moscow (1975).
6. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).
7. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).

DETERMINING THERMAL RESISTANCE OF CONTACT BETWEEN FINISHED WAVY METALLIC SURFACES

V. M. Popov

UDC 536.21

A design formula for determining the thermal resistance of a contact is obtained using functions describing the relief of real wavy surfaces.

Previously completed comparative experimental investigations into the thermal contact resistance (TCR) of metallic contacts between flat-rough and wavy surfaces [1, 2] have established a significant increase in the TCR for the latter for virtually identical grades of surface finish, while an increase in the height of the waves in the surface causes a marked increase in the TCR. At the same time, the theoretical model given in [2, 3] for determining the thermal contact resistance of wavy surfaces is to a certain extent idealized, inasmuch as a homogeneous distribution of the waves by height relative to a standard plane is taken as one of the basic premises. An analysis of profilograph traces from finished metallic surfaces shows that real surfaces represent in most cases a set of waves of a usually spherical or ellipsoidal form with a constant radius subject to a normal (Gaussian) law of distribution by height [4].

Let us examine a contact couple with a wavy surface [2]. In general, the thermal contact conditions presuppose a temperature drop common to all macrocontacts:

$$\Delta T_C = \frac{Q}{2\bar{\lambda}_M a}\varphi.$$

Hence for all macrocontacts the following equality obtains:

$$2\bar{\lambda}_M \Delta T_C = \frac{Q}{a/\varphi}.$$

According to the last equality, the total heat flow is divided as it passes through the individual macrocontacts, i.e., we have the following relation:

$$\frac{Q_1}{a_1/\varphi_1} = \frac{Q_2}{a_2/\varphi_2} = \ldots = \frac{Q_m}{a_m/\varphi_m};$$

hence with an unvariable φ

Voronezh Wood Technology Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 5, pp. 779-785, May, 1977. Original article submitted February 24, 1976.